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Finite-time recurrence analysis of chaotic trajectories in Hamiltonian systems

Chaos **32**, 113144 (2022); <https://doi.org/10.1063/5.0102424> Matheus S. Palmero^{1,2,a},  Iberê L. Caldas¹, and  Igor M. Sokolov²[View Affiliations](#)[View Contributors](#)

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ABSTRACT

In this work, we show that a finite-time recurrence analysis of different chaotic trajectories in two-dimensional non-linear Hamiltonian systems provides useful prior knowledge of their dynamical behavior. By defining an ensemble of initial conditions, evolving them until a given maximum iteration time, and computing the recurrence rate of each orbit, it is possible to find particular trajectories that widely differ from the average behavior. We show that orbits with high recurrence rates are the ones that experience stickiness, being dynamically trapped in specific regions of the phase space. We analyze three different non-linear maps and present our numerical observations considering particular features in each of them. We propose the described approach as a method to visually illustrate and characterize regions in phase space with distinct dynamical behaviors.

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